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An Introduction on New Sets and Functions of Bitopological Spaces

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Abstract

In this paper, we introduce another sort of shut sets in bitopological space (X, s1, s2), utilized it to build new kinds of normality, and introduce new types of continuous function between bitopological spaces. Finally, we demonstrated that our new normality properties are preserved under certain kinds of continuous functions between bitopological spaces.

Keywords: *i-open sets, bitopological space, Normality,*

Introduction

The ideas of ordinary shut, generalized shut (briefly, g-shut), preopen, customary generalized shut (briefly, rgclosed), and generalized preclosed (briefly, gp-shut) sets have been introduced and investigated in. The ideas of preopen sets and ordinary open sets have been reached out to bitopological spaces called ij-preopen and ij-normal open respectively. The mild normality and nearly normality have been introduced in. A powerless type of ordinary spaces has been introduced in called mildly typical spaces. In, the creator utilized the preopen sets to define prenormal spaces, as of late, in the creator has continued the investigation of additional properties of prenormal spaces and furthermore defined and investigated mildly p-ordinary (resp. nearly p-ordinary) spaces which are generalization of both mildly typical (resp. practically typical) spaces and p-ordinary spaces. The idea of generalized preregular shut (briefly, gpr-shut) sets has been introduced in. The idea of binormal spaces has been introduced in. In expanded the ideas of g-shut, gp-shut and rg-shut sets, mildly typical and practically ordinary spaces to bitopological spaces.

In this paper, we expand the idea of gpr-shut sets to bitopological spaces (X, s1, s2) called ij-gpr-shut sets. Likewise, we build another sort of normality in bitopological spaces in view of ij-preopen sets called prebinormal, nearly prebinormal and mildly prebinormal. We utilize the class of ij-gpr-shut sets to characterization these sorts of normality and build new kinds of continuous functions. We demonstrate that the introduced binormality properties are preserved under certain kinds of continuous functions.

All through this paper, the following abbreviations will be embraced: Let A be a subset of a bitopological space (X, s1, s2), the interior (resp. closure) of A regarding topology si (i = 1, 2) will be meant by inti(A) (resp. cli(A)). We mean the arrangement of all shut sets regarding the topology si by i-C(X).

In what follows, let i, j 2 {1, 2} and i „ j.

Definition 1.1. Let (X,τ_1,τ_2) be a bitopological space, a subset A of X is said to be $(\tau_1\tau_2 - i - open\ set)$ if there exists $\tau_1 - open\,set\ \ U \neq \phi, X$ s.t. $A \subseteq \tau_2 - Cl(A \bigcap U)$. The complement of $(\tau_1\tau_2 - i - open\ set)$ is called $(\tau_1\tau_2 - i - closed\ set)$.

Definition 1.2. A bitopological space (X,τ_1,τ_2) is said to be Bi-Topologically Extended for i-open sets ($Bi.T.E.I.$) if $(X, \tau_1\tau_2 - i - open\,sets)$ is a topological space. On the other hand, if $(X, \tau_1\tau_2 - i - open\,sets)$ is not a topological space then, (X,τ_1,τ_2) is called non-Bi-Topologically Extended for i-open sets(not $Bi.T.E.I.$). Where, $\tau_1\tau_2 - i - open\,sets$ denote the family of all i-open sets in the bitopological space (X,τ_1,τ_2).

Example 1.3. Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,\{a\},X\}$, $\tau_2 = \{\phi,\{a\},\{a,b\},X\}$.
$\tau_1 - open\ sets\,are: \phi,\{a\},X$. $\tau_2 - closed\ sets\,are: \phi,\{b,c\},\{c\},X$.
$\{a\} \subset (\tau_2 - Cl(\{a\} \bigcap \{a\}) = X)$, $\{a,b\} \subset (\tau_2 - Cl(\{a,b\} \bigcap \{a\}) = X)$
$\{a,c\} \subset (\tau_2 - Cl(\{a,c\} \bigcap \{a\}) = X)$.
Then, $\{a\},\{a,b\},\{a,c\}$ are $\tau_1\tau_2 - i - open\,sets$.
But, $\{b\},\{c\},\{b,c\}$ are not $\tau_1\tau_2 - i - open\,sets$ because there is no existence $\tau_1 - open\,set\ U$ s.t. $\{b\} \subset (\tau_2 - Cl(\{b\} \bigcap U))$, $\{c\} \subset (\tau_2 - Cl(\{c\} \bigcap U))$
$\{b,c\} \subset (\tau_2 - Cl(\{b,c\} \bigcap U))$ Therefore, $\tau_1\tau_2 - i - open\ sets = \{\phi,\{a\},\{a,b\},\{a,c\},X\}$.
$\tau_1\tau_2 - i - closed\ sets = \phi,\{b,c\},\{c\},\{b\},X$
Where, $(X, \tau_1\tau_2 - i - open\,sets)$ is a topological space. Then, (X,τ_1,τ_2) is a $Bi.T.E.I.$ space.

Example 1.4. Let $X = \{a,b,c,d\}$, $\tau_1 = \{\phi,\{a\},\{b,c,d\}X\}$, $\tau_2 = \{\phi,\{a\},\{c\},\{a,c\},X\}$.
$\tau_1 - open\ sets\,are: \phi,\{a\},\{b,c,d\},X$.
$\tau_2 - closed\ sets\,are: \phi,\{b,c,d\},\{a,b,d\},\{b,d\},X$.
By the same way, in Example 1.3, we have:
$\tau_1\tau_2 - i - open\ sets = \{\phi,\{a\},\{b\},\{c\},\{d\},\{b,c\},\{b,d\},\{c,d\},$
$\{b,c,d\},\{a,b\},\{a,d\},\{a,b,d\},X\}$

$\tau_1\tau_2 - i - closed\ \ sets = \{\phi, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{a,c,d\}, \{a,d\},$
$\{a,c\}, \{a,b\}, \{a\}, \{c,d\}, \{b,c\}, \{c\}, X\}.$

Where, $(X, \tau_1\tau_2 - i - open\, sets)$ is not a topological space. Then, (X, τ_1, τ_2) is not $Bi.T.E.I.$ space.

Definition 1.5. Let (X, τ^i) be a topological space and let A be a subset of X. Recall that the intersection of all i-closed sets containing A is called i-closure of A[6], denoted by $Cl_i(A)$: $Cl_i(A)=\bigcap_{i\in\Lambda} F_i . A \subseteq F_i \quad \forall i$ where, F_i is i-closed set $\forall i$ in a topological space (X, τ^i). $Cl_i(A)$ is the smallest i-closed set containing A.

Definition 1.6. Let (X, τ^i) be a topological space and let A be a subset of X. Recall that the union of all i-open sets contained in A is called i-Interior of A[6], denoted by $Int_i(A)$. $Int_i(A)=\bigcup_{i\in\Lambda} I_i \quad I_i \subseteq A \quad \forall i$. Where, I_i is i-open set $\forall i$ in a topological space (X, τ^i). $Int_i(A)$ is the largest i-open set contained in A.

Theorem 1.7. Every $\tau_1 - open\ \ set$ is $i - open\, set$ in (X, τ_1, τ_2)
Or ($\tau_1 \subset (\tau_1\tau_2 - i -$ open sets)).

Proof Let X be a finite non empty set. Let $\tau_1 = \{\phi, A_1, A_2, ...A_n, X\}$, $\tau_2 = \{\phi, B_1, B_2, ..., B_n, X\}$.

Where, $A_i \subset X, B_i \subset X\ \forall i$.

$\tau_1 - open\ \ sets\, are: \phi, A_1, A_2, ...A_n, X$.

$\tau_2 - closed\ \ sets\, are: \phi, X - B_1, X - B_2, ..., X - B_n, X$.

$\tau_2 - Cl(A_i \bigcap A_i) = \bigcap_{A_i\bigcap A_i \subset F} F$, where F is $\tau_2 - closed\ set$.

At least, X is a $\tau_2 - closed\ \ set$ contains $A_i \bigcap A_i\ \ \forall i$.

Hence, $\tau_2 - Cl(A_i \bigcap A_i) = \bigcap_{A_i\bigcap A_i \subset F} F = X$.

Therefore, $A_i \subset (\tau_2 - Cl(A_i \bigcap A_i) = \bigcap_{A_i\bigcap A_i \subset F} F = X)\ \forall i$.

Then, ($\tau_1 \subset (\tau_1\tau_2 - i -$ open sets)).

The converse of Theorem 1.7 is not true. Indeed, in Example 1.4 $\{b,c\}$ is $\tau_1\tau_2 - i - open\, set$, but is not $\tau_1 -$ open set.■

Definition 1.8. Let (X,τ) be a topological space, recall that extension τ^i [6] is the family of all i-open subsets of space X.

Remark 1.9. [6] (X, τ^i) need not to be a topological space.

Definition 1.10. [6] A topological space (X,τ) is said to be Topologically Extended for i-open sets (shortly T.E.I) if and only if
(X, τ^i) is a topological space. Otherwise is called not T.E.I.

Theorem 1.11. [6] Let X be a non-empty finite set and let $\tau = \{\phi, A, X\}$ where, A is a subset of X and containing only one element. Then, (X, τ) is T.E.I. (i.e. (X, τ^i) is a topological space).

Corollary 1.12. Let (X,τ_1,τ_2) be a bitopological space and let (X,τ_1) be a (T.E.I.) topological space as like as in Theorem 1.11, let $\tau_2 = \tau_1^i$ where, τ_1^i is the family of all i-open sets in a topological space (X,τ_1), then, $\tau_1\tau_2 - i - open\ sets = \tau_2$.

Proof Suppose that $X=\{x_1,x_2,\dots\dots\dots,x_n\}$ and $\tau_1 = \{\phi,\{x_1\},X\}$.
$\tau_1 - open\ sets\ are: \phi,\{x_1\},X.$
By definition of i-open sets, we have:
$\tau_1^i=\{\phi,\{x_1\},\{x_1,x_2\},\{x_1,x_3\},\dots\dots\dots,\{x_1,x_n\},\{x_1,x_2,x_3\},\{x_1,x_2,x_4\},\dots\dots,$
$\{x_1,x_2,x_n\},\dots\dots\dots\dots,\{x_1,x_3,x_4,\dots\dots,x_n\},\{x_1,x_2,\dots\dots\dots,x_n\}=X\}.$
Since, $\tau_2 = \tau_1^i$ then $\tau_2 - closed\ sets\ are: \{x_1,x_2,\dots,x_n\}=X$,
$\{x_2,x_3,x_4,\dots,x_n\},\{x_3,x_4,\dots,x_n\},\{x_2,x_4,\dots,x_n\},\dots\dots,\{x_2,\dots,x_{n-1}\},\{x_4,\dots,x_n\},$
$\{x_3,x_5,\dots,x_n\},\dots,\{x_3,\dots,x_{n-1}\},\dots,\{x_2\},\phi.$
Since, $\{x_1\}$ is the alone $\tau_1 - open\ set \neq \phi, X$ and the intersection between $\{x_1\}$ and the sets $\{x_2\},\{x_3\},\dots,\{x_n\},\dots,\{x_2,x_3\},\dots,\{x_2,x_n\},\{x_2,x_3,x_4\},\dots,$
$\{x_2,x_3,x_n\},\dots,\{x_3,x_4,x_n\},\dots,\{x_{n-2},x_{n-1},x_n\}$ which does not contain $\{x_1\}$, equal to ϕ and by the same way in Theorem 1.11 we have:
$\tau_1\tau_2 - i - open\ sets = \tau_2$ where, $\tau_2 = \tau_1^i$. ■

Example 1.13. Let $X = \{a,b,c\}$,
$\tau_1 = \{\phi,\{a\},X\}$, $\tau_2 = \tau_1^i = \{\phi,\{a\},\{a,b\},\{a,c\},X\}$
$\tau_2 - closed\ sets\ are: \phi,\{b,c\},\{c\},\{b\},X.$
By the same way of the examples mentioned above, we have:
$\tau_1\tau_2 - i - open\ sets = \tau_2$

Definition 1.14. A set A of a bitopological space (X, τ_1, τ_2) is called:

1. $\tau_1\tau_2 - generalized\ closed\ set\ (\tau_1\tau_2 - g - closed\ set)$ [3] if $\tau_2 - Cl(A) \subseteq U$ where $A \subseteq U$ and $U \subseteq X$ is $\tau_1 - open\ set$.
2. $\tau_1\tau_2 - g - open\ set$ [3] if $X - A$ is $\tau_1\tau_2 - g - closed$.
3. $\tau_1\tau_2 - g\,i - open\ set$ if $F \subseteq \tau_2 - Int_i(A)$ where $F \subseteq A \subseteq X$ is $\tau_1 - closed\ set$.
4. $\tau_1\tau_2 - g\,i - closed\ set$ if $X - A$ is $\tau_1\tau_2 - g\,i - open$.
5. $\tau_1\tau_2 - i - star\ genralzed\ closed\ set\ (\tau_1\tau_2 - i * g - closed\ set)$ if $\tau_2 - Cl(A) \subseteq U$ where $A \subseteq U$ and $U \subseteq X$ is $\tau_1 -\ i - open\ set$.
6. $\tau_1\tau_2 - i - star\ genralzed\ open\ set$ $(\tau_1\tau_2 - i * g - open\ set)$ if $X - A$ is $\tau_1\tau_2 - i * g - closed$.
7. $\tau_1\tau_2 - genralzed\ w - closed\ set\ (\tau_1\tau_2 - gw - closed\ set)$[1] if $\tau_2 - Cl_w(A) \subseteq U$ where $A \subseteq U$ and $U \subseteq X$ is $\tau_1 - open\ set$.
8. $\tau_1\tau_2 - genralzed\ w - open\ set\ (\tau_1\tau_2 - gw - open\ set)$[1] if $X - A$ is $\tau_1\tau_2 - gw - closed$.

In the following example X is a finite set.

Example 1.15. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ $\tau_2 = \{\phi, \{a\}, X\}$.

From definitions mentioned above we have:

Conclusion

In this paper, τ1τ2-¯g-open sets were introduced in the bitopological spaces and their properties were studied. Further, their properties were contrasted and a portion of the corresponding generalized open sets in the overall topological spaces and bitopological spaces.

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